

# Testing Human Capital theory: a case study of Canada within the World

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## **Abstract**

Human capital had evolved macroeconomic models removing the single use of TFP when modeling world income disparities. A recent development from Manuelli and Sehadri had suggested that human capital and its evolution can be measured within an income generation framework. Similarly to Ben Porath's models, their model uses returns to human capital and to goods as well as ability and fertility rates, but in addition incorporates years of education and wages. The purpose of this paper is to test the suitability of Manuelli and Seshadri's theory on the Canadian context, first I used their model to estimate total factor productivity and check its ranking across provinces and territories of Canada. Then I return to the global context and attempt to validate the model by comparing estimated variables with their observed (or estimated elsewhere) counterparts. Finally, I test the sensitivity of the model's parameters, to identify to which parameters the modeler shall pay more attention. It was found that returns to human capital and to goods played a vital role. I attempted to estimate their values from world data, only to find that returns to goods seems stable across the world at around 0.37 to 0.40, while world returns to human capital vary largely between countries with world average at 0.14 and Canada or USA values at 0.48, suggesting that human capital used to accumulate more human capital

is key from a development perspective and that countries should encourage learning throughout the life of productive individuals.

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**Keywords:** Human Capital, Income, Sensitivity, Validation

### **Introduction:**

There is a vast amount of literature concerned with the large disparities of wealth (income, output) or development across countries in the world (Hall and Jones 1999, Quah 1996, Freyer 2008). Historically, research on such disparities has built upon two main blocks: technological change (and what it drives it) and human capital. Both build upon modified versions of the Solow's model (1956) and the latter uses some sort of Mincer specification (1974) for validation. Differences in income (or output) have been typically explained by either the role of human capital or total factor productivity. Both approaches have been developed upon such factors and they differ in whether to abandon or expand the neoclassical model based upon Solow (1956) after the expansion done by Mankin, Romer, and Weil (1992). The first method looks into technology to explain differences on total factor productivity and output. The other one focuses on the role of human capital (Hendricks 2002). The main drawbacks of the technology (TFP) approach is the lack of endogenous capability to model individual choices on elements related to human capital formation and technological implementation. The problem of the second revolves around difficulties on measuring human capital stocks typically addressed by using a Mincer specification (Mincer 1974).

It was Hendricks (2002) the first one to suggest that both approaches should not be contradictory but rather complementary. Recent developments hinged on the use of an explicit human factor production function (Klenow and Rodriguez-Clare 1997) to explicitly model the role of education quantity and quality in building human capital stocks and of those in total factor productivity and on income disparities. At the core of such models one finds the aggregation of decision makers selecting the number of years of

education and its quality. One of such works is that of Erosa, Koreshkova and Restuccia (2009) which revolves on measuring investment on schooling (goods) to estimate quality and then to observe the role of TFP to explain cross country differences on income per capita. Another one is that of Manuelli and Seshadri (2014) who follow a similar approach with quantity and quality on the core but with the claim of having pinned down the functional form of a human capital production function which in turn is used to estimate required differences in TFP to explain across country income disparities (output per worker). Manuelli and Seshadri's model contains schooling time (quantity), post-schooling training (quality of skills) and age earnings profile and finds a less dramatic role of TFP as found by others before.

**TABLE 1. Human capital and TFP on economic development**

Author / year	Model	Findings	Human Capital	T.F.P.
Solow (1956)	Decreasing returns to scale to capital. Savings and population growth are exogenous.	<b>Steady-state</b> level of <b>income-per-capita</b> . Conditional convergence, countries reach different steady states of <b>income-per-capita</b>	NO	NO
Mincer (1974)	Statistical regression	Returns of education and experience can be used to <b>estimate human capital stocks</b> . Largely used for calibration.	NO	NO
Mankin, N.G., Romer, D. and Weil, D.N. 1992	Added accumulation of Human Capital (H) to Solow's model	Interactions of human capital and savings ( $s$ ) and population growth ( $n$ ) with income ( $Y$ ).	YES	NO
Klenow, P.J. and Rodriguez-Clare, A. (1997)	<b>Human capital production function.</b> Primary/secondary schooling-attainment. School quality to produce measures of human capital.	Cross-country productivity differences explain over 50% of level differences of GDP per worker (1985 data). Differences in productivity growth explain growth rate differences of GDP	NO	YES
Hall and Jones (1999)	Relies on Solow's residual (TFP)	Human and physical capital on output per worker. Social infrastructure explains differences across	NO	YES

		countries		
Hansen, G.D. and Prescott, C.P. 2002.	Overlapping generations, <b>maximize profits and utility. Role of Technology on output</b>	From stagnation to growth (production) as land is outshined by human and capital growth (more skilled labor)	YES	NO
Hendricks (2002)	<b>Human capital accumulation and total factor productivity</b>	Used immigrants to estimate differences on human capital	YES	YES
Erosa, Koreshkova and Restuccia (2009)	<b>Human capital investments</b> (schooling time & expenditure on schooling, use goods as a measure of quality).	Relative importance of time versus good inputs used to explain human capital and see role of TFP as an amplification factor for disparities	YES	YES
Manuelli and Seshadri (2014)	Schooling quantity, training (quality). Role of TFP is smaller than previously predicted	Created Theoretical estimation of <b>Human capital production function</b> and used to estimate required TFP to explain countries differences	YES	YES

## 1.1 Problem Statement

Even though conceptually Manuelli and Seshadri's model has the advantages aforementioned, the model was calibrated only to the United States through five moments, and there is a lack of validation of this framework: it is uncertain how well it predicts observed variables across countries, how sensitive it is to its parameters, and if its applicability can be extended to regions within a country or across countries of the world.

## 1.2 Objective

The objective of this paper is to study the applicability of Manuelli and Seshadri's model: (1) by validating the model prediction capabilities, (2) by testing the model sensitivity to its parameters and, (3) by testing its applicability within regions of a given country or across countries of the world.

### 1.3 Method summary

This paper studies the suitability of a recent development to explain income/output differences per worker for regions within a country, in a case study for Canada. This paper employs the model specification recommended by Manuelli and Seshadri (2014) and studies the suitability of such model by: (1) analyzing the sensitivity of the model's parameters, (2) comparing its results with those predicted by other authors and (3) estimating the values of the most sensitive parameters from world data. Some interesting inferences were withdrawn from the analysis.

### 1.4 The model

This section describes the model, for a more detailed explanation the reader is directed to Manuelli and Seshadri (2014). The aim here is to explain the mechanisms built within the model in order to serve as a preamble for the validation/calibration strategy. The model (Equations 1 to 2) generates human capital stocks through an income maximization problem (expanded Ben Porath 1967). The individual maximizes discounted value of net income by selecting the fraction of time (at a given age) used to accumulate human capital ( $n(a)$ ). This amount equals one (100% of the time) up until the end of formal school (age =  $6+s$ ) which depends on each country. The individual also chooses the amount of early childhood (up to the age of 6) investments ( $x_E$ ) and the amount of market inputs to produce human capital up until current age ( $x(a)$ ). It assumes the same technology for human capital accumulation during schooling and training (on the job).

$$\max \int_6^R e^{-r(a-6)} [wh(a)[1 - n(a)] - x(a)da] - x_E - \eta(s)$$

(1)

$$\dot{h}(a) = z_h [n(a)h(a)]^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a) \quad \text{and} \quad h(6) = h_E = h_B x_E^{\nu}$$

(2)

The solution to the problem is restricted to a law of motion of human capital that considers its depreciation  $\delta_h$ , market inputs  $x(a)$  and fraction of

time  $n(a)$  used to acquire human capital in the current period and the individual's innate learning skills  $z_h$ . In addition to individual and accumulated human capital stocks, the solution to the problem estimates years of schooling and an earnings profile per country (output per worker) given by Equations 3 and 4.

$$\frac{h_B^{1-\gamma}}{z_h^{1-\nu} w^{\gamma_2 - \nu(1-\gamma_1)}} = m(6 + s)^{1-\nu(2-\gamma)} e^{(1-\gamma)(\delta_h + r\nu)s} \left(\frac{\nu}{r + \delta_h}\right)^{-(1-\gamma)\nu} \left(\frac{\gamma_1^{1-\gamma_2} \gamma_2^{\gamma_2}}{r + \delta_h}\right)^{1-\nu} \left[ 1 - \frac{r + \delta_h}{\gamma_1} \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_2 r + \delta_h(1-\gamma_1)} \frac{1 - e^{-\frac{\gamma_2 r + \delta_h(1-\gamma_1)}{(1-\gamma_2)} s}}{m(6+s)} \right]^{\frac{(1-\gamma)(1-\nu(1-\gamma_1))}{(1-\gamma_1)}} \quad (3)$$

Where  $m(6 + s) = 1 - e^{-(r + \delta_h)(R - 6 - s)}$

$$\hat{y}(s, p) = (1 - \nu) w \left[ \frac{z_h \gamma_1^{1-\gamma_2} \gamma_2^{\gamma_2}}{r + \delta_h} \left(\frac{(1-\tau)w}{p_w}\right)^{\gamma_2} \right]^{\frac{1}{(1-\gamma)}} \left\{ e^{-\delta_h} h(6 + s) \frac{r + \delta_h}{\gamma_1} \int_{6+s}^{p+6+s} e^{-\delta_h(p+6+s-t)} m(t) dt - \frac{\gamma}{\gamma_1} m(a) \frac{1}{(1-\gamma)} \right\} \quad (4)$$

The model explicitly considers wages ( $w$ ) and allow individuals to be more willing to invest in education if their level of skills is higher, it also acknowledge the importance of school quality as oppose to quantity. The other explicit elements on the model are the demographics of the country. The number of individuals at a given age and time  $N(a, t)$  is estimated from population growth rate  $\eta$  and life-terminal age  $T$  as shown in Equation 5.

$$N(a, t) = e^{-\eta t} \left( \eta \frac{e^{-\eta a}}{1 - e^{-\eta T}} \right) \text{ where } \eta = \frac{f}{B} \quad (5)$$

As in any other model, several coefficients need to be calibrated:  $\gamma_1$  returns to labor,  $\gamma_2$  returns to input markets,  $\delta_h$  depreciation,  $\eta$  population growth rate,  $p_E$  price of early childhood inputs,  $p_w$  cost of on the job-training among others. A relative wage rate for skilled workers was computed based on Hendricks (2002) in order to estimate wage variation and compare it to

the age earnings profile proposed by Manuelli and Seshadri (2014).

## Calibration

The model developed by Manuelli and Seshadri is locally calibrated to Canadian provinces. Parameters values from the original model (for the US) and from its calibration to Canada are presented in Table 1.

**TABLE 1 Calibration Values and parameters**

Variable / Parameter	US	CAN	CAN (model)
Wage rate 55/25	2	2.29	2.29
Years of schooling	12.05	about 13	13.2
Schooling / GDP	4.5%	about 5%	4.76%
Early / GDP	<b>1.1%</b>	<b>Less than US%</b>	<b>0.88%</b>
Income ratio (64/55)	0.79	0.76	0.786
Retirement Age (T)	<b>78.5</b>	<b>81</b>	<b>81</b>
Model's coefficient $\gamma_1$	0.486	Not observed	0.4875
Model's coefficient $\gamma_2$	0.400	Not observed	0.404

*Value of preprimary investment for US is 0.4 and for Canada is 0.2% of GDP*

Canada as a country was used as the benchmark for the normalization of provinces and territories; in the original model (Manuelli and Seshadri 2014) the US was used as the benchmark economy and countries around the world compare to it. Canadian provinces and territories were sorted out based on output per worker. Beginning of working age for Canada was set to 25 years, life expectancy to 81 years and fertility rate to 1.66.

## Results

The model predicted relative total factor productivity for provinces and territories in Canada (Table 2). I also used Hendricks (2002) model to estimate relative wage rates for skilled labor (I assumed 67% were skilled workers). Table 2 presents the demographic data and output per worker used to estimate TFP and the relative wage.

**TABLE 2 Demographics and estimated values for Canada's Provinces or Territories**

Province/Territory	Life expectancy	Fertility rates	Output per worker	TFP	Relative wage rate (Hendrick)-skilled
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<b>Prince Edward Island</b>	80.5	1.63	0.7	<b>0.87</b>	<b>0.15</b>
<b>Nova Scotia</b>	80	1.48	0.81	<b>0.87</b>	<b>0.17</b>
<b>New Brunswick</b>	80.5	1.52	0.82	<b>0.88</b>	<b>0.18</b>
<b>Quebec</b>	81	1.69	0.84	<b>0.88</b>	<b>0.18</b>
<b>Manitoba</b>	79.5	1.96	0.9	<b>0.89</b>	<b>0.20</b>
<b>British Columbia</b>	82	1.52	0.96	<b>0.91</b>	<b>0.21</b>
<b>Ontario</b>	81.5	1.57	0.98	<b>0.90</b>	<b>0.21</b>
<b>Yukon</b>	75	1.58	1.12	<b>0.91</b>	<b>0.25</b>
<b>Saskatchewan</b>	79.5	2.03	1.13	<b>0.91</b>	<b>0.25</b>
<b>Alberta</b>	81	1.9	1.43	<b>0.95</b>	<b>0.33</b>
<b>Nunavut</b>	75	2.97	1.46	<b>0.95</b>	<b>0.34</b>
<b>Newfoundland- Labrador</b>	79	1.46	1.47	<b>0.95</b>	<b>0.34</b>
<b>Northwest Territories</b>	75	2.11	1.99	<b>0.99</b>	<b>0.48</b>

1.4.1 The ranking of Canadian provinces and territories from a total factor productivity and relative wages seems to match that expected, resource based provinces with lower population are more productive and individuals earn higher wages.

#### 1.4.2 Sensitivity Analysis

1.4.3 The value of parameter  $\gamma_1$  and  $\gamma_2$  were explored, only one parameter at the time was changed by holding constant all others. As seen on Tables 3 and 4. Small variations of either parameter produced large impact on the calibration targets, therefore a one percent change was chosen for the sensitivity analysis of  $\gamma_1$  and  $\gamma_2$ . As seen on Table 3 a 1% change in  $\gamma_1$ , implies a 10.5% change for the wage rate 55/25, a 8.56% on the number of years of schooling, and a 13.6% on the early-education expenditure and 9.24% on the schooling to GDP.

**TABLE 3 Sensitivity to  $\gamma_1$**

Variable / Parameter	-1%	Model CAN	+1%
<b>Wage rate 55/25</b>	2.05	2.29	2.679
<b>Years of schooling</b>	12.07	13.2	14.45
<b>Schooling / GDP</b>	4.32%	4.76%	5.23%
<b>Early / GDP</b>	1.0%	0.88%	0.78%
<b>Income ratio (64/55)</b>	0.786	0.786	0.786
<b>Model's coefficient <math>\gamma_1</math></b>	0.482625	0.4875	0.492375
<b>Model's coefficient <math>\gamma_2</math></b>	0.404	0.404	0.404

**TABLE 4 Sensitivity to  $\gamma_2$**

Variable / Parameter	-1%	Model CAN	+1%
<b>Wage rate 55/25</b>	2.12	2.29	2.535



<b>Years of schooling</b>	12.39	13.2	14.10
<b>Schooling / GDP</b>	4.41%	4.76%	5.12%
<b>Early / GDP</b>	0.986%	0.88%	0.79%
<b>Income ratio (64/55)</b>	0.787	0.786	0.786
<b>Model's coefficient <math>\gamma_1</math></b>	0.4875	0.4875	0.4875
<b>Model's coefficient <math>\gamma_2</math></b>	0.39996	0.404	0.40804

Manuelli and Seshadri claimed that the model exhibit variations according to the fertility rate. A base rate of 1.66% and variations of 40% is presented in Table 5. As seen neither income ratios nor years of schooling are affected, the larger variations are observed for early childhood and schooling investments as percentages of GDP.

**TABLE 5 Sensitivity to  $\eta$**

<b>Variable / Parameter</b>	<b>-40%</b>	<b>Model CAN</b>	<b>+40%</b>
<b>Wage rate 55/25</b>	same	2.29	same
<b>Years of schooling</b>	same	13.24	same
<b>Schooling / GDP</b>	4.72%	4.76%	4.81
<b>Early / GDP</b>	0.877%	0.88%	0.897%
<b>Income ratio (64/55)</b>	same	0.786	same
<b>Fertility rate <math>\eta</math></b>	1.00%	1.66%	2.33%

Variations to total factor productivity (TFP) and level of innate ability ( $Z_h$ ) were explored: a 10% on TFP approximately corresponded to variations of 5% on  $Z_h$  as shown on Table 6 and 7.

**TABLE 6 Sensitivity to  $z_h$  ability**

<b>Variable / Parameter</b>	<b>-5%</b>	<b>Model CAN</b>	<b>+5%</b>
<b>Wage rate 55/25</b>	1.92	2.29	2.77
<b>Years of schooling</b>	11.76	13.24	14.42
<b>Schooling / GDP</b>	4.27 %	4.76%	5.16
<b>Early / GDP</b>	1.11%	0.88%	0.73%
<b>Income ratio (64/55)</b>	same	0.786	same
<b>Zh ability</b>	0.3173	0.334	0.3507

**TABLE 7 Sensitivity to TFP**

<b>Variable / Parameter</b>	<b>-10%</b>	<b>Model CAN</b>	<b>+10%</b>
<b>Wage rate 55/25</b>	1.93	2.29	2.81

<b>Years of schooling</b>	11.78	13.24	14.49
<b>Schooling / GDP</b>	4.33 %	4.76%	5.12
<b>Early / GDP</b>	1.07%	0.88%	0.75%
<b>Income ratio (64/55)</b>	same	0.786	same
<b>TFP</b>	0.9	0.334	1.1

In summary (Table 8), the model is very sensitive to  $\gamma_1$  and  $\gamma_2$ , a 1% change in  $\gamma_1$  and  $\gamma_2$  requires about 10% changes in TFP or 5% changes in innate ability ( $z_h$ ) to produce similar results. Hence, a large effort should be concentrated in estimating the values  $\gamma_1$  and  $\gamma_2$ , Solow (1956) suggested that the success of modeling lies in the model mechanisms being capable of abstracting by much the phenomena at hand without being heavily affected by the parameters. Hence in this sense, Manuelli and Seshadri (2014) model fails unless a reliable approach for the estimation of returns to scale in the human capital production function, that is, the role of labor in production of human capital  $\gamma_1$  and the role of goods in the production of human capital ( $\gamma_2$ ) could be found. Total factor productivity and  $z_h$  impact more early investment as ratio of GDP values so could be used to adjust such variable.

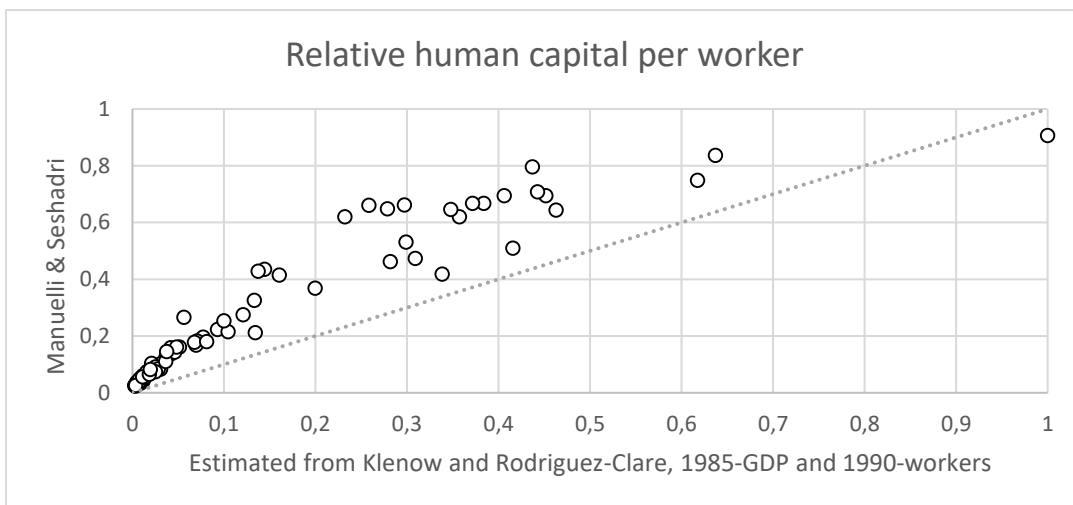
**Table 8. Sensitivity summary**

<b>Variable / Parameter</b>	<b>5% <math>Z_h</math></b>	<b>10% TFP</b>	<b>1% <math>\gamma_1</math></b>	<b>1% <math>\gamma_2</math></b>	<b>40% <math>\eta</math></b>
<b>Wage rate 55/25</b>	16.2%	15.7%	10.50%	7.40%	Same
<b>Years of schooling</b>	11.1%	11%	8.56%	6.13%	Same
<b>Schooling / GDP</b>	10.3%	9.03%	9.24%	7.35%	0.84%
<b>Early / GDP</b>	26.1%	21.6%	13.60%	12.05%	0.4%

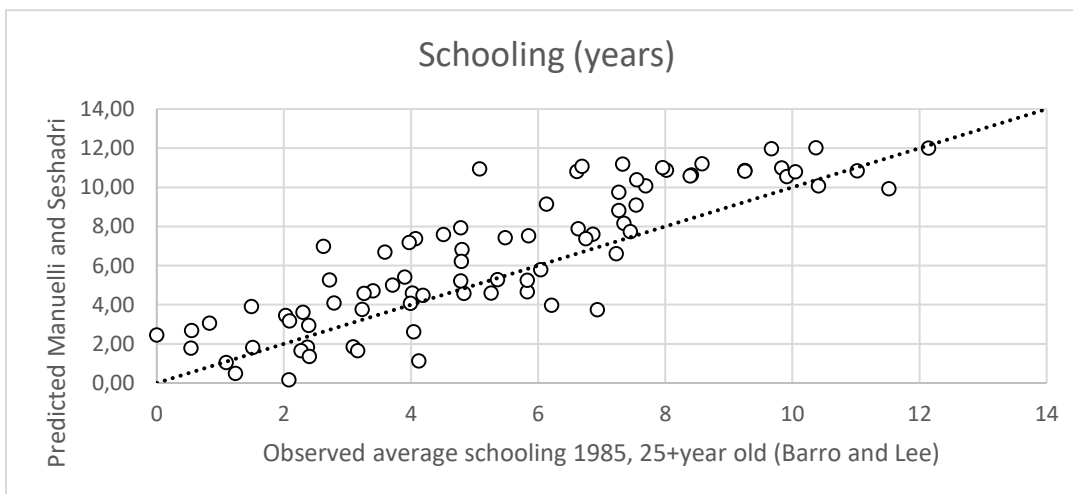
#### **1.4.4 Comparison of predicted versus observed results**

We built several indicators from observed data and compared them with those predicted by Manuelli and Seshadri's model. The only exception is that of *Human capital per worker* which was constructed from the estimation of human capital per output based on 1985 data by Klenow and Rodriguez-Clare (1997). Data used for this comparisons included gross domestic product (GDP) for the year 1985 and 1990, expenditure of education on GNI and GNI for the year 1990 and labor force data (number of workers) for the year 1990 all from the World Bank. Manuelli and Seshadri's

(2014) model was used to estimate the same indicators. All graphs plot the 45 degree perfect-equivalence reference-line. The results of relative human capital shown on Figure 2 over-predict human capital as predicted by Klenow and Rodriguez-Clare (1997). Schooling showed a better spread around the one to one equivalence but was also slightly overestimated.



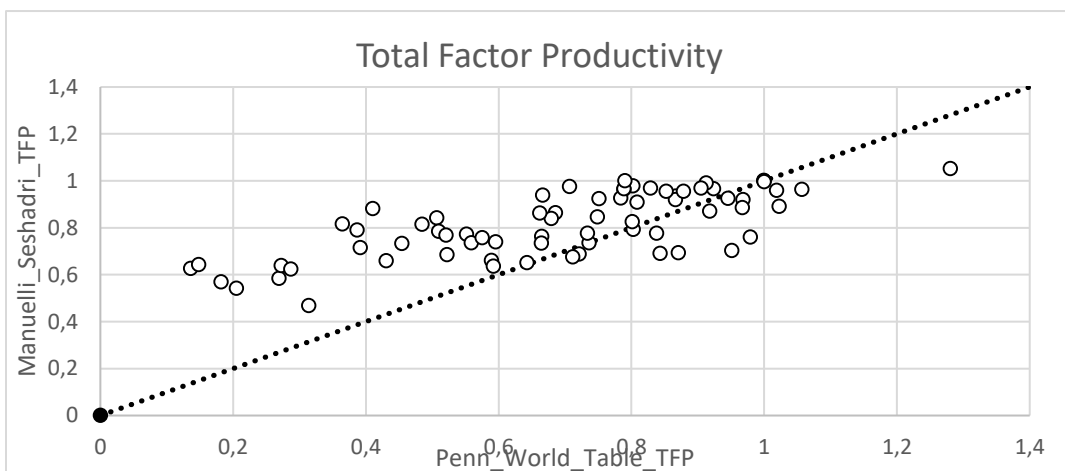
**Figure 2. Relative Human Capital per worker,  $H_{US} = 1$**



**Figure 3. Schooling in number of years**

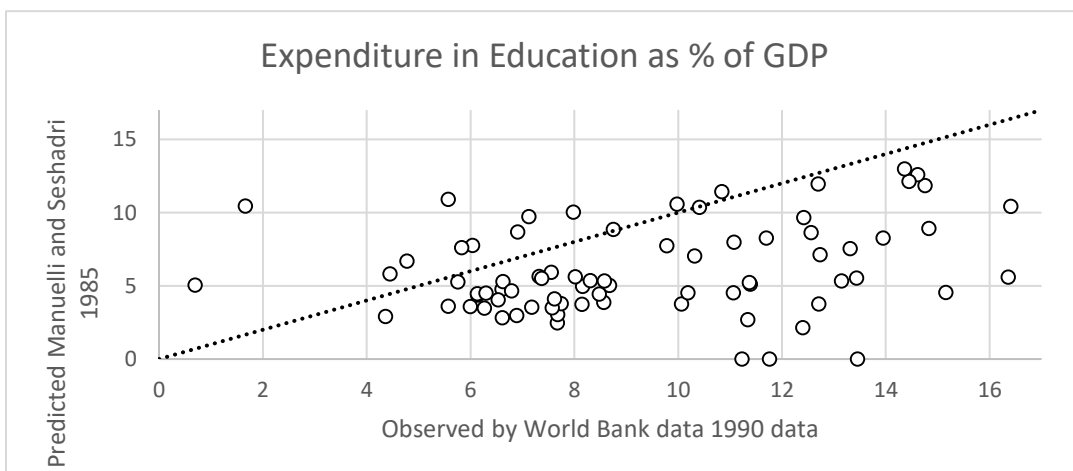
Penn World table 8.0 contains estimates of total factor productivity, which

have been compared to those estimated by Manuelli and Seshadri in Figure 4. Clearly Manuelli and Seshadri overestimates the values presented by the Penn World Table 8.0.



**Figure 4. Total Factor Productivity**

A large spread was observed when comparing observed investment in education as percentage of GDP with predicted values, however, we built observed expenditure from expenditure as ratio of GNI and had to bring it back in terms of GDP with 1990 data, 1985 data was unavailable.



**Figure 4. Expenditure in Education as % of GDP**

### Estimation of parameters

The ultimate goal of the model (Manuelli and Seshadri 2014) was to

estimate human capital, in that respect and as compared to the estimates from Klenow and Rodriguez-Clare (1997), the model overestimates such values, the problem is that human capital is a variable we do not observe and hence once we track down the model capability in terms of years of schooling or expenditure in education as percentage of GDP, the model does a better job. The question now turns to the model ability to be calibrated to replicate world data. As it turns out, one of the key features of the model is its heavy reliance on the values of  $\gamma_1$  and  $\gamma_2$ . Is there a method to learn the value of such parameters from the data? As it turn out a Full Bayesian regression guided by a Markov-Chain-Monte-Carlo (Gibbs-Sampling) with a non-informative prior could do this job. However, current model is highly dimensional and several attempts to estimate it using commercial software (OpenBUGS) failed. A simplified version of the model -presented below- was used for such a purpose.

A simplified Ben-Porath (1967) law of motion of human capital in steady state (Equation 6 instead of Equation 2) along with first order condition for  $x$  (Equation 7), resource constraint (Equation 8) and first order condition for schooling (Equation 9) from an income maximization problem similar to the one in Equation 1 were used to estimate the returns to human capital  $\gamma_1$  and from investment in education  $\gamma_2$  from data of 81 countries of the world. Equation 10 presents the FOC w.r.t.  $n$ .

$$\delta^h h = z_h [nh]^{\gamma_1} x^{\gamma_2} \quad (6)$$

$$\frac{1}{\beta} = 1 + H_1 - \delta^h \quad \text{Where } H_1 \text{ is the FOC of Equation 6 w.r.t. } n \quad (7)$$

$$c + x + \delta^k k = y \quad (8)$$

$$\frac{H_1}{H_2} = (1 - \tau)w \quad \text{Where } H_2 \text{ is the FOC of Equation 6 w.r.t. } h \quad (9)$$

$$H_1 = z_h \gamma_1 n^{\gamma_1} h^{\gamma_1 - 1} x^{\gamma_2} \quad (10)$$

I solved for three unknowns: the amount of time spend acquiring human capital ( $n$ ), the investment in goods for human capital ( $x$ ) and the amount of human capital ( $h$ ).

From Equation 7 and taking the derivative one can find Equation 11

which contain three unknowns  $n$ ,  $h$  and  $x$ . Take now the derivative of Equation 11 with respect to  $h$  and use Equation 9 to obtain Equation 12 which only contains two unknowns, take now Equation 6 and plug in Equation 12 to obtain Equation 13, finally take Equation 8 and plug it into Equation 13 which solves for  $n$ . The system given by Equations 11 to 13 can be used to solve for the amount of human capital (Equation 15).

$$\frac{\frac{1}{\beta} - 1 + \delta^h}{z_h} = \gamma_1 n^{\gamma_1} h^{\gamma_1} x^{\gamma_2} \quad (11)$$

$$h = (1 - \tau)wn \quad (12)$$

$$n = \frac{z_h x^{\gamma_2}}{\delta^h (1 - \tau)^{1 - \gamma_1} w^{1 - \gamma_1}} \quad (13)$$

$$n = \frac{z_h (y - \delta^k k - c)^{\gamma_2}}{\delta^h (1 - \tau)^{1 - \gamma_1} w^{1 - \gamma_1}} \quad (14)$$

$$h = \frac{z_h (y - \delta^k k - c)^{\gamma_2}}{\delta^h (1 - \tau)^{-\gamma_1} w^{-\gamma_1}} \quad (15)$$

I concentrate the attention now to the estimation of  $\gamma_1$  and  $\gamma_2$ . A full Bayesian estimation using OpenBUGS (reference) was run to estimate their values from the observed data, a non-informative prior was used to learn from the data the probabilistic distribution for the 95% CI of the values of  $\gamma_1$  and  $\gamma_2$ . Values of human capital estimated by Manuelli and Seshadri's model were used on the left-hand-side of Equation 15 and values of observed output per capita, capital per capita and consumption per capita were used to build human capital and the system was target with estimating the two unknown parameters as stochastic nodes. Income taxes were estimated for the countries and both depreciation rates were set to 0.075.

Figure 5 illustrates the model used with 55 countries for which data was available. Human capital per worker was obtained from Klenow and Rodriguez-Clare (1997), data for labor income tax was fixed it to 0.3 for all countries (this needs to be revised in future research), physical capital per worker and output per worker were obtained from the world bank database, consumption was fixed to 80% of output for all countries. Results from the estimation are also shown on Figure 5. As seen an estimation of the values of

$\gamma_1$  and  $\gamma_2$  from 55 countries of the world yields very dissimilar results than those estimated by Manuelli and Seshadri (2014). Value of  $\gamma_1$  observes a huge discrepancy ( $E(\gamma_1)=0.1452$  versus  $g_1=0.48$ ) meanwhile values of  $\gamma_2$  are much closer ( $E(\gamma_2)= 0.3729$  versus 0.4 in the original model).

## Conclusion

Manuelli and Seshadri's model predictions seems to match well productivity of Canadian provinces and territories. In a world context their model seems to accurately replicate observed years of education and to overstate relative human capital per worker as compared to classical specifications. Their model is very sensitive to returns to human capital and to goods ( $\gamma_1$  and  $\gamma_2$ ). Returns to goods ( $\gamma_2$ ) across countries of the world does not seem to vary much. Returns to human capital invested to produce more human capital vary largely; the world average does not suggest that such return contributes as largely as observed in the US or Canada, this results seems to align with the belief that quality of the education plays a very significant role even more in developed countries and that the impact of education on income ranges a lot among countries.

```
model {  
  zh <- 0.334  
  dk <- 0.075  
  dh <- 0.075  
  tao <- 0.3  
  for(i in 1 : 55) {  
    h[i] ~ dnorm(mu[i],tao) #H= human capital per worker  
    mu[i] <- zh*pow(y[i]-dk*k[i]-c[i],g2)*pow((1-tao)*w[i],g1)/dh  
  }  
  sigma <- sqrt(1/tao)  
   $\gamma_1$  ~ dnorm(0,0.010)  
   $\gamma_2$  ~ dnorm(0,0.010)  
  tau ~ dgamma(0.001, 0.001)  
}
```

```
list(γ1=0.1, γ2=0.7,tau=0.001) #chain initialized with prior for γ1=0.1 and γ2=0.7
list(γ1=0.7, γ2=0.1,tau=0.001) #chain initialized with prior for γ1=0.7 and γ2=0.1
```

	mean	sd	MC_error	val2.5pc	median	val97.5pc	
<b>start</b>							
<b>sample</b>							
γ1	0.1452	0.08712	1.647E-4	0.01273	0.1342	0.3438	1
2204000							
γ2	0.3729	0.02942	5.594E-5	0.3057	0.3765	0.4199	1
2204000							

**Figure 5. Full Bayesian Model and estimated returns to human capital and to goods**

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